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What Mathematical Knowledge Do Prospective Teachers Reveal When Creating and Solving a Probability Problem?

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Abstract: The teaching of probability is conditioned by teachers' mathematical knowledge. In this paper, an exploratory study is carried out with prospective teachers. A training task was designed requiring them to create and solve a probability problem using the values of euro coins, which was adapted to students aged 11 to 12. The study aimed at determining what mathematical knowledge prospective teachers show when dealing with the task. The data were collected through the Moodle online Campus. We framed the data analysis in the Mathematical Knowledge for Teaching model and we used content analysis as the methodological approach. The results indicate that, despite finding evidence of adequate common and specialised mathematical knowledge, in approximately half of the prospective teachers participating in the study, too many of them still show a lack of knowledge in both subdomains. There was also little evidence of knowledge of the curriculum. The main finding of the research is that, when prospective teachers get involved in complex creative tasks, they mobilised together specialised and common mathematical knowledge, working into different mathematical processes such as problem posing and solving, communication, and argumentation, which reinforces the need to continue working on these types of complex tasks.

Keywords: common content knowledge; probability; prospective teachers; specialised content knowledge; task design

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1. Introduction

Probability has been part of the educational programmes at secondary and university levels for a long time in many countries, and some of them have also introduced probabilistic content in the elementary school syllabus [1,2]. That introduction is justified based on the great importance of understanding the language of chance in our daily lives, as random phenomena permeate our lives in many ways [1–7]. Notions such as probability or uncertainty appear frequently during our adult life, for example, when we calculate forecasts of medical, financial, or environmental risks, when we study the reliability of a product, or when we make weather predictions. It is, therefore, necessary to deal with the language of chance (to distinguish impossible, sure, more or less likely, and rank them) and probability, as the mathematical tool that quantifies and measures degrees of beliefs about chance.

However, probability was included in primary school curricula not before the late 1980s, after the National Council of Teachers of Mathematics [8] introduced "Data and Chance" in the Curriculum and Evaluation Standard for School Mathematics, until the late 1980s. This means that the incorporation of probability at primary school level has been gradual since the early 1990s in many countries. The Principles and Standards for School Mathematics [9] strengthen this idea by indicating the need for compulsory education programmes to prepare students to deal with data analysis and probability, including knowledge of these areas from the age of 3 years old onwards. The approach promoted by this institution, and which has been incorporated into the curricula of a large number of countries, is an experimental approach, which facilitates the approach to stochastic experience from childhood [10]. Countries such as Australia or Ireland already introduce probability in their primary school curricula. Although it is true that in Ireland probability is not taught until the age of 9 [11], in Australia it is taught from the age of 6 [12]. In both cases, the emphasis is on promoting the language of chance by using vocabulary from everyday life, related to uncertainty, and being able to identify the results of simple random experiments. If we focus on the Spanish curriculum, the inclusion of probability is even more recent, [13], being introduced in 2006, while statistics were added at the beginning of the 1990s [10,14]. The official primary education syllabus [15] incorporated probability in a block of content entitled "Statistics and Probability". It gradually incorporates probability through activities in which students must identify and distinguish random experiments from deterministic ones, know how to determine whether the occurrence of an event is more or less probable or make predictions about the possibilities of obtaining an outcome in simple random experiments, such as games of chance [16].

In Spain, there is a tendency to have a formalistic knowledge of mathematics which means that PTs have difficulties in teaching probability in schools [2]. In addition, probability contents are included in the last block of contents of the Spanish curriculum, with these topics at the end of the textbooks. This means that the content is often not taught due to a lack of time and, if it is taught at all, it is usually done quickly and mechanically, without generating deeper reasoning and learning [17]. However, the importance of probabilistic literacy for children is underlined in [7]. Moreover, probability literacy [6,7] is closely related to literacy in other areas of mathematics, such as arithmetic and statistics. The significant increase in problematic situations involving uncertainty, both in daily life and in professional contexts, requires mathematical reasoning to help understanding and addressing them, producing citizens who are committed, constructive and reflective [18] and, thus, empowering them in the use of probability in context situations, is one of the current goals of elementary education [2,19]. In this sense, and in order to guarantee a more contextualised teaching of probability, it is necessary to promote the use of authentic tasks, "understood as those tasks that simulate approaches to real life in the most reasonable way possible, beyond that of solving decontextualized exercises in a mechanical way and using formulas that can be applied without being understood" [2] (p.5).

Within the described context, it is obvious that receiving adequate training in probability is essential for prospective teachers (hereafter PT or PTs if plural). This training must include mathematical and didactical aspects so that not only cognitive aspects of probability are learnt but also that PTs are able to identify latent concepts in a didactic situation related to probability. Therefore, PTs will help their pupils acquire probability literacy at the end of primary school through the acquisition of an appropriate basic probabilistic language that allows identifying random phenomena and situations, making hypotheses, and estimating and calculating simple probabilities, within a problem-solving context that acts as the backbone of the subject [7,20,21].

However, and presumably due to the recent incorporation of probability in the Spanish curriculum, PTs' training in probability is still scarce. Many authors indicated that both in-service teachers and PTs, show deficiencies in their probability knowledge, even when they had received prior training [16,22]. In [23–28] it is indicated that PTs present equiprobability biases, incorrect probabilistic calculations and lack of combinatorial reasoning that prevent them from correctly determining the sample space, among other issues. Moreover, PTs also suffer from the same biases as primary school students [27,29–32]. In this sense, previous research [16,22,24,27,28,33,34] showed that it is necessary to reinforce specialised content knowledge during PTs' initial training since they generally showed a lack of both mathematical and didactical knowledge. Moreover, in [28] it is stated that "some teachers are unable to identify latent concepts in a didactic situation related to statistics or probability" (p. 87).

The goal of this research is to analyse PTs' mathematical and pedagogical knowledge when they create, solve, and justify their selection of a probability problem, by using Laplace's definition. Therefore, we propose a study in which PTs are asked to create and solve a probability problem adapted to the sixth grade of primary school (11–12 years old), by using a detailed task [35,36]. Thus, we pose the following research question: "What kind of knowledge do PTs show when designing a probability problem in a specific school context?"

2. Theoretical Framework

Probabilistic literacy refers to a citizen's ability to be able to understand and interpret daily life probabilistic information [2,7,37,38]. For [7], when we talk about probability literacy, we must consider probability as a basic tool for statistics to be able to study more advanced topics, preparing us to face the increasing number of random and chance events that we may encounter in our daily lives [38]. Probabilistic literacy includes both cognitive and dispositional components [7]. These elements are shown in Table 1.

Table 1. Cognitive and dispositional elements, from [7].

Cognitive Elements	Dispositional Elements
Big Ideas How to calculate probabilities Language Context Critical questions	Critical stance Beliefs and attitudes Personal feelings regarding uncertainty

PTs will, therefore, mobilise both cognitive and dispositional elements when working on probability in the classroom. In this paper, we will focus on the cognitive elements, paying special attention to probability calculation, language, and context.

On the other hand, when dealing with probability at elementary school it is always necessary to consider the different meanings of the term probability, as indicated in [39]: intuitive, experimental, classical, subjective, or axiomatic. In this paper, we will focus on the classical meaning of probability established by Laplace, which is the most frequent in Spanish textbooks and resources, so that it is often the only meaning that teachers and students deal with [27]. The classical meaning of probability is taken from Laplace's seminal work "Essai philosophique sur les probabilités" (1814). Laplace defined the probability of an event A in a random experiment as the ratio between the number of favourable outcomes to the occurrence of A and the number of possible outcomes. This formula can be used whenever the sample space is finite, and the elemental events are equally likely.

At elementary school, the main application of Laplace's definition is the calculus of probabilities of simple events in games of chance or similar contextualized situations. Although this type of game is familiar to children, mobilising their combinatorial reasoning for calculating probabilities is often difficult [34], and not only for children but also for PTs [33]. This lack of combinatorial knowledge implies difficulties in the classroom when dealing with the classical meaning of probability.

In order to analyse the mathematical knowledge about probability revealed by PTs, we will consider the Mathematical Knowledge for Teaching model (MKT) [40]. This model emerges from the ideas of Shulman [41], an author who revolutionised how teachers' knowledge of a given teaching discipline was conceptualised. Shulman proposed the separation of teachers' knowledge into content knowledge, considered as the knowledge properly studied within the disciplinary settings, and pedagogical content knowledge, understood as the pedagogical knowledge that a teacher must have to teach the content of the discipline. In this sense, [42] explains that there is "an area in the intersection of general pedagogical knowledge and subject matter knowledge that represents a teacher's capacity to develop and implement instruction on a particular content in particular ways that provides the greatest potential for enhanced student understanding" (p. 16). By following that structure, the MKT model distinguishes two domains: Mathematical Knowledge (MK) and Pedagogical Content Knowledge (PCK) (see Figure 1).

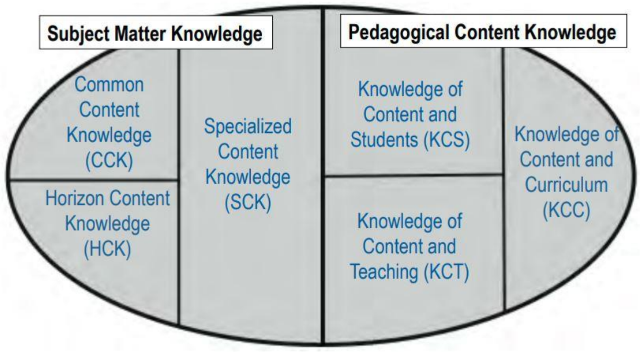

Figure 1. Mathematical Knowledge for Teaching (MKT) [43] (p.15).

On the one hand, teachers' MK is divided into three subdomains:

- Common Content Knowledge (CCK): this refers to mathematical knowledge in situations that are not exclusive to teaching. Within the probability context, the CCK includes applying definitions and properties (such as Laplace's definition), correctly operating with fractions, decimals, and probabilities, making a proper combinatorial analysis of the random experiment, or solving contextualized probability problems, among others. For instance, if we flip two coins, we can wonder what is the probability of getting one head and one tail: after properly creating the sample space, we can identify which are the possible outcomes and which are the favourable ones, so that, by applying Laplace's definition, we determine the probability.
- Specialized Content Knowledge (SCK): this is the exclusive knowledge for teaching mathematics. It refers to different types of knowledge: the adequate selection of tasks for working on certain contents or educational levels, the appropriateness of explanations about a concept, the justification of properties, methods, or algorithms, or the understanding of students' productions. If we focus on probability at elementary school, the following will be SCK: knowing how to perform combinatorial counts, selecting an appropriate task for using Laplace's definition, or explaining why the repetition of a random experiment can provide an estimate of the probability of a related event.
- Horizon Content Knowledge (HCK): this refers to how mathematical topics relate to each other. An example of HCK in the considered contexts is the connection between fractions, decimal numbers, and probabilities, as described in [16].

On the other hand, the PCK is divided into three subdomains:

- Knowledge of Content and Students (KCS): this refers to the interactions between the content and how students think and learn or their previous knowledge, their difficulties and mistakes, main obstacles, etc. In the case of probability, KCS refers to the usual biases in probabilistic reasoning, the way in which students handle randomness or how to help students when making specific mistakes in probability.
- Knowledge of Content and Teaching (KCT): this refers to the knowledge linked to teaching a content, such as building teaching processes adapted to students, or deciding which students' contributions are desirable. In this context, KCT concerns designing didactic sequences to teach probability, such as choosing appropriate representations to deal with Laplace's formula at the elementary school.
- Knowledge of Curriculum (KC): this refers to the knowledge of curricular objectives, goals, orientations, artefacts, assessment, etc. It is sometimes referred also as Knowledge of Content and Curriculum. In the case of probability, KC consists of the knowledge of curricular guidelines, contents, and assessment criteria about probability in primary education, as referred to in the introduction.

Previous studies using the MKT model [24] to analyse PTs' mathematical knowledge in probability showed that the lack of CCK also produced a lack of SCK. It was also observed how PTs showing evidence of CCK had a lack of SCK [34]. According to our knowledge, a combination of evidence of SCK and lack of evidence of CCK has not been found in any previous research. This seems quite logical because if there was a lack in PTs' CCK it would be difficult to have the required knowledge to teach unfamiliar, difficult, or confusing mathematical concepts.

On the other hand, most of this previous research applied closed questionnaires for analysing PTs' knowledge, but it is not common to consider the probability knowledge in the context of problem-posing and problem-solving, while this type of detailed task is suggested by authors such as [35], who understands that: "some of the professional tasks of the mathematics teacher are to choose, design and sequence the mathematically relevant tasks for the learning of their students" (p. 33). Designing mathematical tasks allows PTs to develop their competence in educational analysis [36], and it is useful to determine what their mathematical knowledge is like, and where to reinforce the instruction if needed. The task proposed in this work aims to analyse PTs' content and pedagogical mathematical knowledge through the creation of a probability problem under certain constraints, as well as its solving process.

3. Method

In this section, we will describe the population and research subject, the instrument and the data analysis process conducted in this study.

3.1. Population and Research Subject

The population consisted of the 168 PTs enrolled in the subject "Mathematics and its Didactics III" in the third year of the Bachelor's Degree in Primary Education at the University of Oviedo. This subject involves contents of statistics, probability and problem-solving and it is an annual course. The participant PTs were informed about their participation in a research project and gave their consent.

The PTs were asked to undertake a task, which was carried out completely online under Moodle. We obtained 111 answers (66.07 % response rate). Two of the answers were discarded for being blank. Therefore, the research subject consisted of n = 109 PTs' answers.

3.2. Instrument

Based on [35], PTs were asked to design and solve one probability problem, adapted to the sixth grade of elementary school (11–12-year-old children). In addition, the PTs had to explain their creative process and justify how their problems fitted the standards of the given educational level. Therefore, PTs must mobilise their CCK about combinatory and probability to propose a problem that can be solved by using Laplace's definition. Additionally, by requesting PTs to justify their choice of problem, explaining their creative processes, and justifying the alignment with the sixth grade syllabus, we intended to obtain evidence of their SCK and KC. Nevertheless, since MKT subdomains cannot be interpreted as separated compartments, other clear evidence about the rest of MKT subdomains were also pointed out. In this way, we aimed to identify the strengths and weaknesses of the PTs in the development of their task, so that we could decide in which aspects we needed to reinforce our instruction to improve their training in the field.

The statement proposed by the researchers [44] (p. 107) for the task was the following:

Within Euro currency, coins can be: 1, 2, 5, 10, 20, or 50 cents, and 1 or 2 euros.

Starting with this situation: "In my pocket, I have 3 coins . . . ", add constraints so that you can formulate a problem for sixth graders in which the question is: "What is the probability of having more than 2 euros in my pocket?" and the answer must be one number between 0.6 and 0.75 (both included).

The task is not so simple, both from the didactical and the mathematical points of view. PTs must mobilise their PCK and MK simultaneously to carry out the task. We briefly address some mathematical issues about the task. Without any constraint about the value of the three coins, the result could be any combination of three of the different types of coins (1, 2, 5, 10, 20, and 50 cents, and 1 and 2 euros), which means there would be 120 possible combinations. To determine the number of favourable cases to have more than 2 euros in the pocket, we would need to determine (by using a tree diagram or a similar strategy) that, due to the type of coins, it is necessary to have at least two 1-euro coins or one 2-euro coin. Thus, the number of favourable cases without any limitation is 43. Therefore, not adding restrictions to the original statement would produce a problem that clearly overpasses sixth grade, since combinatory at that level only concerns simpler cases determined by the multiplication principle. That is why, to adapt the problem to sixth grade level, PTs should add constraints, for example, by prefixing some of the coins in the pocket. Adapting the task to sixth grade means that more in-depth specialised knowledge is required. The CCK in probability, combined with the SCK, that allows realising that the problem without constraints would be too much difficult for sixth grade, will determine the effectiveness of the task for the selected level with the specified mathematical content. Firstly, the sample space should be reduced from the original 120 cases to achieve a more affordable sample size for being handled. On the other hand, to obtain a solution belonging to [0.6, 0.75] a high rate of favourable cases is needed. Knowing the value of some of the coins in the pocket also could ease to achieve a solution, thus, by thinking about this process, PTs will activate their CCK in probability, and the process of adapting the problem to the required level will reveal their SCK and KC.

3.3. Data Analysis Process

In a previous work [44], we detailed the process of classifying PTs' productions into correct, partially correct, and incorrect problems, each with intermediate subclassifications. In the current article, by using a directed content analysis, in which each PT was a unit of analysis [45], we combine the classification in [44] with the analysis of the CCK, SCK and KC subdomains of the MKT model.

In order to conduct the analysis under the CCK, SCK and KC subdomains of the MKT model, different aspects were considered. Firstly, to find evidence of CCK, the statement and its resolution were taken into account. Evidence of high levels of CCK was considered to be those problems that were correctly formulated and mathematically well solved. Errors in the problem statement or problem-solving were considered as a lack of CCK. When the problem was not solved, we could consider it as a lack of evidence of CCK. In other words, expressions and statements that made explicit reference to the mathematical content of the problem were considered as evidence of CCK. For example, it is evidence of CCK if they explain that "to obtain the probability value, it is necessary to apply Laplace's definition which is the number of favourable outcomes divided by the number of possible outcomes. This definition can be applied because there is a finite number of elementary events that are equiprobable". Secondly, in order to find evidence of SCK, the adaptation of the problem to the educational level and explanations regarding the appropriateness of the problem were also taken into account. Evidence of SCK was considered to be explanations indicating that the statement proposed by the researchers needed to include restrictions, so that the sample space was more manageable, as well as explanations concerning the use of Laplace's definition since this is the main content studied in the sixth grade of primary school in Spain. This last explanation also referred to PTs' KC, since this is the content that appears explicitly in the primary education curriculum for this educational level in Spain. Thus, we consider expressions and statements that made explicit reference to the way of adapting the problem to the educational level as evidence of SCK. For example, we consider it evidence of SCK if they explain that "it is necessary to reduce the number of possible outcomes because if there were many possible outcomes, the sample space will be very difficult to obtain for sixth graders". Finally, we also consider expressions and statements that made explicit reference to the content of the primary education curriculum for the fixed level as evidence of KC. For example, we consider it evidence if they explain that "the problem can be solved by using Laplace's definition because it is part of the contents of the curriculum for sixth graders". Aspects that, although not explicitly reflected in a detailed explanation, could show PT's knowledge of the mathematical, pedagogical or curriculum content, or lack of it, were not considered as evidence but as signs of the correspondent knowledge.

To combine the categories found in [44] with the results of the analysis of the three aforementioned subdomains of the MKT model, we considered those aspects that referred to whether the problem was mathematically well solved and whether the problem was adapted to the requested educational level. In this way, we were able to classify correct, partially correct, and incorrect problems according to whether they were well or poorly solved mathematically, and whether or not they were adapted to the educational level requested. KC was also taken into account in all the categories obtained in the classification, since it did not seem to show a direct relationship with them, as we found little evidence of it, and it seemed to be distributed between the PTs' answers: those who were able to adequately deal with the task and those who did not solve it so satisfactorily.

The analysis was done separately by two of the researchers, and then the differences were discussed with all four researchers to find a consensus. By using this method, we can present MKT evidence more concisely. Evidence of all these subdomains was found; despite not being the focus of this study, other subdomains that may appear during the analysis of the data were also considered, such as evidence of HCK, for example when some PTs explained the relationship between the calculation of probabilities and the proper treatment of fractions.

4. Results

By combining the categories obtained in [44] with the analysis of the MKT subdomains, we obtained five types of students' productions, which are described and analysed in detail in Table 2.

Table 2. Types of students' productions under the MKT lens.

Types of Students' Productions	*n*	%
Mathematically correct problems corresponding to sixth grade of primary education and adapted to the wording model in which the PTs show evidence of deep CCK and SCK	34	31.2%
Mathematically correct problems corresponding to sixth grade of primary education, but which do not follow the proposed wording model, in which the PTs show evidence of deep CCK and SCK	25	22.93%
Partially correct or incorrect problems in which the statement could correspond to sixth grade of primary education but in which the PTs show evidence of a poor CCK because they do not solve the problems properly	19	17.43%
Partially correct problems not corresponding to sixth grade of primary education, in which the PTs show evidence of poor SCK	1	0.92%
Partially correct or incorrect problems not corresponding to sixth grade of primary education, nor correctly solved in which the PTs show evidence of lack of CCK and SCK	30	27.52%

Note that no specific reference is made to KC in Table 2. This is due to the limited evidence of KC found.

4.1. Mathematically Correct Problems Corresponding to Sixth Grade of Primary Education and Adapted to the Wording Model, in Which the PTs Show Evidence of Deep CCK and SCK

In these types of answers, there was evidence of a deep CCK and SCK. Thirty-four PTs were able to create a mathematically correct problem and properly solve it, and both the problem and its solving were adapted to the specified educational level set and fitted to the proposed statement model. Regarding KC, evidence was only shown in six PTs' productions, when they justified their election based on the curricular content.

Example 1(a) shows a statement classified in this category (PTs' productions examples are codified with a number representing the student and, if necessary, a letter denoting the corresponding excerpt).

Example 1(a): *In my pocket, I have 3 euro coins. These can be 2 euros, 50 cents and/or 20 cents; these values can be repeated for more than one coin. What is the probability of having more than 2 euros in my pocket?*

The statement proposed in E1(a) was valid and appropriate for sixth grade because the sample space was considerably reduced to 10 possible cases so that the problem became affordable to sixth graders. The PT exhaustively defined the sample space, analysing all possible combinations and selecting the favourable cases. Finally, by applying Laplace's definition, she obtained the probability value, as shown in E1(b).

Example 1(b): *To apply Laplace's definition I must know the number of favourable outcomes and the total number of possible outcomes. I already know the total number of possible outcomes (there are 10). Six of them are favourable outcomes:*
1st combination: 20 cents, 20 cents, 20 cents
2nd combination: 20 cents, 20 cents, 50 cents
3rd combination: 20 cents, 20 cents, 2€ (favourable outcome)
4th combination: 50 cents, 50 cents, 2€ (favourable outcome)
5th combination: 50 cents, 50 cents, 50 cents
6th combination: 50 cents, 50 cents, 20 cents
7th combination: 2€, 2€, 2€ (favourable outcome)
8th combination: 2€, 50 cents, 2€ (favourable outcome)
9th combination: 2€, 2€, 20 cents (favourable outcome)
10th combination: 2€, 20 cents, 50 cents (favourable outcome)
P (A) = favourable outcomes/ possible outcomes
P (having more than 2 euros in my pocket) = 6/10 = 0.60

Thus, this PT's production provides evidence of a deep CCK. Her justification about why she chose this statement was also very detailed.

In E1(c) we can see an excerpt showing clear evidence of deep SCK, by warning that, in case of considering all the coins, the problem would be very difficult for the addressed educational level. Furthermore, she also evidenced a combination of CCK and SCK by acknowledging the need of reducing the size of the sample space to make the problem more approachable:

Example 1(c): *If the 3 coins could take on the 8 existing values (2 euros, 1 euro, 50 cents, 20 cents, 10 cents, 5 cents, 2 cents and 1 cent) the possible number of combinations that could be made would be too large to work with sixth graders (as requested); I decided to reduce these possibilities.*

Evidence of KCT was also found when trying to adapt the problem to the capabilities of sixth graders as shown in excerpt E1(d).

Example 1(d): *To keep it simple and taking into account the grade at which the statement is intended, I reduced the possible values for the combinations to 3.*

Moreover, as reflected in E1(e), she also showed evidence of deep SCK when she considered that the fewer combinations in her problem, the more appropriateness for sixth graders ("more tractable").

Example 1(e): *Thus, there are 10 combinations, as I mentioned above. This is a significantly smaller number and therefore a more tractable number.*

In this PT's answer, there was no explicit reference to the curriculum. However, based on her explanations, we could find signs of KC as she worked with the specific contents of the level. Moreover, she indicated (see E1(f)) that students could start experimenting with higher level contents without using formulas.

Example 1(f): *In this way, they [primary school students] will be able to experiment with combinations, even if they do not know the formula that determines the number of combinations.*

4.2. Mathematically Correct Problems Corresponding to Sixth Grade of Primary Education, but Which Do Not Follow the Proposed Wording Model, in Which the PTs Show Evidence of Deep CCK and SCK

There were twenty-five problems classified in this section. The statement did not fit the requested model, but there was evidence of deep CCK and SCK. Only two of them also showed some evidence of KC. Example 2(a) shows a problem of this type:

Example 2(a): *Marcos went to the bookshop to buy a new comic. His mother gave him 3 euros, but the comic costs between 5 and 6 euros. In his pocket he still has three coins, which can be 2 euros, 50 cents or 20 cents. What is the probability that Marcos has more than two euros in his pocket?*

Although it did not follow the requested structure for the task, E2(a) is an appropriate problem for the requested educational level. The followed solving process was canonical: it started with the creation of the sample space, then the selection of the favourable and possible outcomes, and ended with the application of Laplace's definition to obtain the probability value obtaining the correct solution which also fitted with the interval as shown in E2(b).

Example 2(b): *Sample space:*
2€ - 2€ - 2€
2€ - 2€ - 50 cent
2€ - 50 cent - 50 cent
50 cent - 50 cent - 50 cent
50 cent - 50 cent - 20 cent
50 cent - 20 cent - 20 cent
20 cent - 20 cent - 20 cent
20 cent - 20 cent - 2€
20 cent - 2€ - 2€
2€ - 50 cent - 20 cent
Following Laplace's definition to calculate the probability of Marcos having more than 2 euros in his pocket we divide the number of favourable outcomes (the ones in bold type) by the number of possible outcomes (sample space).
P (A) = number of favourable outcomes / numbers of possible outcomes = 6/10 = 0.6. This is the probability of Marcos having more than 2 euros in his pocket to pay for the comic.

This PT showed evidence of KC when she mentioned the contents on probability for the considered level of primary education, such as the use of Laplace's definition, as we can observe in the excerpt E2(c):

Example 2(c): *The statement is in accordance with the contents on probability taught in primary education; it allows me to implement Laplace's definition and to establish the sample space concerning the question asked. It may be a bit simple, but it allows both to review what has been learnt about probability and to begin an explanation of the topic.*

4.3. Partially Correct or Incorrect Problems in Which the Statement Could Correspond to Sixth Grade of Primary Education but in Which the PTs Show Evidence of a Poor CCK Because They Do Not Solve the Problems Properly

There were nineteen problems (ten partially correct and nine incorrect) classified in this category. They showed that PTs presented an evident lack of CCK, as they were unable to solve the problem posed by themselves. However, there were some signs of SCK, since the statements were correct and adapted to sixth grade, even though they were not correctly solved. Excerpt E3(a) shows an example:

Example 3(a): *I have 3 euro coins in my pocket, but I don't know exactly what they are worth. The only thing I am sure about is that none of the coins are worth less than 20 cents since I have put them in my piggy bank. Bearing this in mind, what is the probability of having more than 2 euros in my pocket?*

The statement proposed in E3(a) was approachable. However, the solving process was inadequate, showing a lack of CCK by incorrectly defining the sample space, since repeated outcomes were considered, making a total of 64 outcomes, when the real number of possible combinations was 20 outcomes.

Only two out of the ten partially correct problems showed evidence of KC. We can see one example of this in the justification of the problem in example E3(b), in which the PT proposed the use of appropriate solving strategies:

Example 3(b): *I consider it to be adequate because it is solved with strategies suitable for sixth graders since I have used a tree diagram for solving it and LaPlace's [sic] definition, which is the definition par excellence of probability that students use.*

4.4. Partially Correct Problems Not Corresponding to Sixth grade of Primary Education, in Which the PTs Show Evidence of Poor SCK

We found only one production in which, despite the posed problem and its solving procedure being mathematically correct, there was a clear lack of SCK, since the PT was not aware that the level of the statement proposed was too high for sixth graders. The statement itself was written in a quite murky way to be understood by a sixth grader, and that made it even more difficult to solve it, as can be seen in E4(a):

Example 4(a): *Miguel has 3.88 euros in his wallet, divided into euro coins that are never repeated, but his trousers have no pockets, so he decides to take only 3 coins to go and buy 4 packs of stickers at the kiosk. He doesn't remember the value of the coins he took, the only thing he remembers is that none of them was copper-coloured and he also remembers that they added up to more than 1 euro. If each pack of stickers costs 0.50 euros: What is the probability that he has more than 2 euros in his pocket?*

Additionally, in excerpt E4(b) we can also find a clear lack of mathematical knowledge when talking about "logarithms" instead of "algorithms".

Example 4(b): *I think it is appropriate because it is not limited to the simple repetition of a logarithm to solve the problem posed. We can know the logarithms involved in the problem, but if we do not understand the statement, it will not be possible to continue successfully.*

4.5. Partially Correct or Incorrect Problems Not Corresponding to Sixth Grade of Primary Education nor Correctly Solved in Which the PTs Show Evidence of Lack of CCK and SCK

There were thirty problems classified in this category (mostly incorrect problems), that showed a clear lack of CCK or SCK. Either the statements were poorly posed or meaningless, or the problem solving was mathematically incorrect (or both). Different examples of this classification are presented. In excerpt E5, no restrictions were imposed, and the proposed solving process made no sense. Without constraints, the sample space would be extremely large, which makes the problem unsuitable for the sixth grade. Moreover, there were many different possible solutions, as it was not clear what we had to calculate. On the other hand, the computation of the probability is meaningless, as the problem is only about checking the combinations. All these facts revealed a clear lack of CCK and SCK by the PT.

Example 5: *"In my pocket, I have three euro coins..." and I want to know what they might be. Knowing that the probability of having more than 2€ is between 0.6 and 0.75, how can I find it out?*

Solving:

First of all, the statement asks us to calculate the probability of having more than 2€, so we start with this amount and assume that one of the coins is 2€. For the other two, we randomly select two small amounts, such as 10 and 50 cents.
We know that, since we have three coins, the probability of picking each one is 0.33 (1/3). If we have to obtain a probability that is between the values 0.6 and 0.7, we can see that 0.33 × 2 results in 0.66, a value that is valid for solving this problem.
From this point we can proceed in two ways:
(a) P (draw a coin different from two) = 1 − 0.33 = 0.66
(b) P (have in your pocket the 2€ coin and another coin) = 0.33 + 0.33 = 0.66

Let us consider now Example 6(a):

Example 6(a): *In my pocket, I have 3 euro coins, they can be of: 1, 2, 5, 10, 20 or 50 cents or 1 or 2 euros. I know that one of the coins will be either 1 or 2 euros and that there is also the option that it could be one of the six cent coins mentioned above. Taking into account the two aforementioned conditions, we would obtain from both of them, using a mathematical operation, the number of possible outcomes, what is the probability of having more than 2 euros in one's pocket?*

Under the conditions proposed in E6(a), the PT worked with all the possible coins, so, in practice, no restriction was added to the original problem. Moreover, the solution provided was not correct, as it did not consider all the possible outcomes in the sample space, as shown in E6(b). Again, this evidenced a lack of CCK and SCK.

Example 6(b): *There are a total of 43 favourable outcomes. Following the instructions from the statement:*
- *One of the 3 coins will be 1 or 2 euros: 64 options*
- *To have cents coins: there are 6 different cent coins*
- *From those 2 conditions, we obtain the number of possible outcomes: 64 + 6 = 71*

P (A) = number of favourable outcomes/number of possible outcomes = 43/71 = 0.6056.
So, 0.61 is the probability of having more than two euros in the pocket.

Finally, let us look at production E7(a):

Example 7(a): *In my pocket, I have three euro coins and five marbles, I know that the coins are two euros and fifty cents, but I don't know how many of each. I make a bet with my friends in which, by putting my hand in my pocket only twice, I have to prove that I have more than two euros. What is the probability of having more than two euros in my pocket? I also know that the outcome is between 0.6 and 0.75.*

The statement in E7 introduced marbles without explaining that we have to use them to solve the problem. In addition, there were several spelling and punctuation mistakes (in the original Spanish version) making it difficult to understand some parts of the explanation. The PT's resolution did not make any sense, making many mistakes, as shown in E7(b). The problem would be complicated for sixth graders which, again, showed a clear lack of CCK and SCK.

Example 7(b): *I reach into my pocket twice and the first time I pick up a 50 cent coin, the second time I can get another 50 cent coin, a 2€ coin or a marble. But my main interest is the 2€ coin to win the bet.*
Now I would make a tree diagram in which one side would be the coins, which are two eighth parts, and from this side would come another two sides for each coin left in the bag, each of them having a probability of 0.5.
The second side would be the marbles, which would be six-eighths of a probability.
To solve it we would calculate two tenths (probability of the two coins) times one half (probability of the 2€ coin, which is the one we are interested in) plus six tenths (probability of getting a marble).
(0.2 × 0.5) + 0.62 = 0.72.

5. Discussion and Conclusions

The goal of this research was to analyse the PTs' MKT when they create, solve, and justify their selection of a probability problem, with a strong basis on combinatorial thinking and the involvement of Laplace's definition for determining the probability. After the analysis of the subdomains of the MKT, particularly CCK, SCK and KC, we have observed that more than half of the PTs (59 out of 109) were able to create and properly solve a problem that was aligned with the sixth grade syllabus, that is, the problems were mathematically correct and adapted to the educational level, using a manageable sample space for sixth graders as well as using Laplace's definition to solve the problem. Therefore, we found strong evidence of both CCK and SCK, and signs of KC, as shown in example E1(a–f). The first main finding is obtaining such a percentage (54.13%) of PTs who showed evidence of deep knowledge in both CCK and SCK, at the same time, which was not usual in previous large-scale studies [24,25] like this one (even though the sample size of this study is considerably greater than previous studies in a similar context). In our opinion, the greater percentage of PTs' evidencing CCK and SCK at a high level is due to the instrument. In previous research on mathematical knowledge of PTs about probability [23–28], the instrument did not provide as much qualitative information as the one used here about the creative process in designing a task, ability in problem-solving and the argumentation in the justification of the task, as other studies are limited to the use of questionnaires without using authentic tasks involving PTs [46–55].

When considering those PTs who did not follow the requested wording model but provided a different type of problem which was properly solved and explained, we consider this as evidence of SCK, because they were able to develop a correct problem and to solve it adequately, moreover their problems were appropriate for sixth grade. On the other hand, the fact that they did not fit the requested problem model could be viewed as a lack of CCK, but also as an issue or lack of attention. Nevertheless, it would need to be discussed how far CCK includes a proper and careful reading comprehension of a given wording as part of mathematical knowledge or as a general literacy. Whatever these mistakes are considered, a second important finding of this study leans on these examples, because they show how SCK can be mobilised despite not being endorsed by a correct CCK.

Despite the good results achieved, it is still very worrying that for almost half of PTs we observed a lack of CCK or SCK and even both. This result confirms [31] regarding the weak PTs' knowledge of probability and its didactics. In that group of PTs' productions, the main obstacle in CCK about probability was the combinatorial reasoning, since many PTs failed in determining the counting of favourable or total possible cases, as in [24,27,31,56]. This lack of combinatorial reasoning also results in PTs who did not know that the sample space of the problem they were posing was very difficult for sixth graders, which makes the problem a higher level. This implies a lack of SCK, also related to the lack of CCK. In addition, these findings underline that already described in [34] about low levels of CCK combined with lack of evidence of SCK, suggesting the need to emphasize both subdomains in teacher training.

The research literature [24,52] also described that the lack of CCK usually implies a lack of SCK. It seems expected that a lack of evidence of CCK would imply a lack of evidence of SCK since a teacher without mathematical knowledge would not know how to teach it. However, in the present study, we found a considerable group of 19 PTs who showed evidence of SCK, by creating valid and adequate problems, but a lack of CCK, because they did not know how to solve them properly, and this is the third main finding of the study. We think that these PTs could be influenced by the formulation of the task, which was quite predetermined (with a fixed beginning and ending, and the PTs were asked to add restrictions), and which helped in thinking of a statement aligned with the sixth grade syllabus, even when the PTs were not able to solve their own problem, due to lack of CCK.

On the other hand, to adapt the problem to the sixth grade syllabus, in addition to their CCK and SCK, PTs used their KC, but we have a very limited number (less than expected) of evidence of that use. We assume that perhaps they have mobilised their KC, but they considered that too obvious to make it explicit. However, we do not have enough data supporting this fact. We have also noted that PTs' explanations about the creative process and the alignment to syllabus were extremely brief and concise, even poor, and, in many cases, absent or without any relevant information. This is consistent with [57], who already pointed out that PTs found it difficult to provide mathematical explanations, especially explaining how they carried out a task or a proof. Our initial aim was to analyse the subdomain SCK by asking for that argumentation, but it could not be observed in nearly any of the answers. Thus, even when asking the PTs to justify the followed steps could enhance metacognitive processes, we checked that this ability was barely demonstrated in PTs' productions, such as [57] demonstrates.

From these findings, some conclusions can be obtained. First, there is interest in using detailed and authentic tasks in the case of probability. There is a lot of literature about complex mathematical tasks with students, but few studies deal with using detailed probability tasks in PT training. The task presented here involves the mobilization of, at least, CCK, SCK and KC, and it could also mobilise other subdomains of MKT. Moreover, if we consider the mathematical processes described in [9], this task obviously involves problem-solving, but also reasoning, proof and argumentation (PTs must argue about the procedure they designed), communication (PTs must explain their reasoning and show how their problem is aligned with the sixth grade syllabus), intra-mathematical connection (with combinatory and number decomposition, to obtain different ways to sum more than two euros with three coins), and changes of representation (transition from verbal to probabilistic language) or transnumeration [58] which is the process of translating between data representations. This task allowed us to identify the strengths and weaknesses of PTs regarding probability and its didactics. The use of the MKT model to analyse PTs' mathematical knowledge for teaching has allowed us to determine where we should focus our teacher training, helping us to identify the challenges we need to address in future training programmes and allowing us to improve our own instruction, as also indicated in [59]. Studies such as [60] show that probability continues to be taught in a decontextualised way, becoming arithmetically based and dominated by the direct application of formulas, so we consider it necessary to continue working in this direction in order to improve prospective teacher training. Second, there is a need to reinforce connections between different knowledge subdomains of PTs' knowledge, considering them together during the initial teacher training. In this sense, we can conclude that considering common and specialised knowledge as insulated watertight compartments is not an efficient way for developing PTs' mathematical knowledge, since PTs could attain acceptable levels in one of these subdomains but not in the others. This reinforces the idea that we should continue working with tasks such as the one presented in this study, which mobilises both mathematical knowledge and specialised knowledge.

The main limitation of this work comes from non-random sampling in a single university, which prevents us from generalising our findings. However, the sample size considered in this work is considerably greater than in other exploratory and qualitative analyses in the literature of PT training in probability. A second limitation comes from the structure of the instrument, since PTs were asked to compose a problem with fixed starting and ending, which could limit their creativity. Therefore, we plan to redesign the task and propose a more open one, so that we can find out if those cases showing evidence of SCK but lack of CCK (which is one of the unexpected results of the paper) are related to the structure of the task.

Before carrying out the experiment, we believed that the PTs' explanations on how they created the problems and why they chose that problem could help us to identify evidence of SCK and KC. However, the few explanations provided by the PTs had little relevance. Therefore, we intend to introduce a second change in the task, so that we help the PTs in constructing more consistent argumentations.

Finally, as another future line of research, we plan to replicate the experiment with PTs from other universities in which the training programmes separate the subjects of probability and its didactics. In this way, we could compare the results and determine whether a joint training of probability and its didactics influences the results or not.

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